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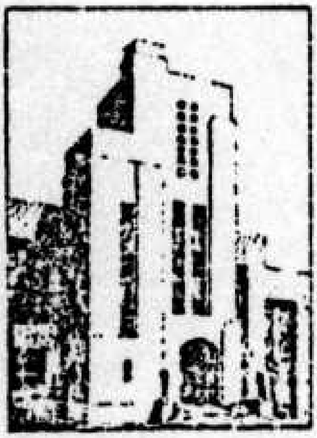
PREDICTION OF DYNAMIC STABILITY DERIVATIVES OF AN ELONGATED BODY OF REVOLUTION

by

L. Landweber and J.L. Johnson



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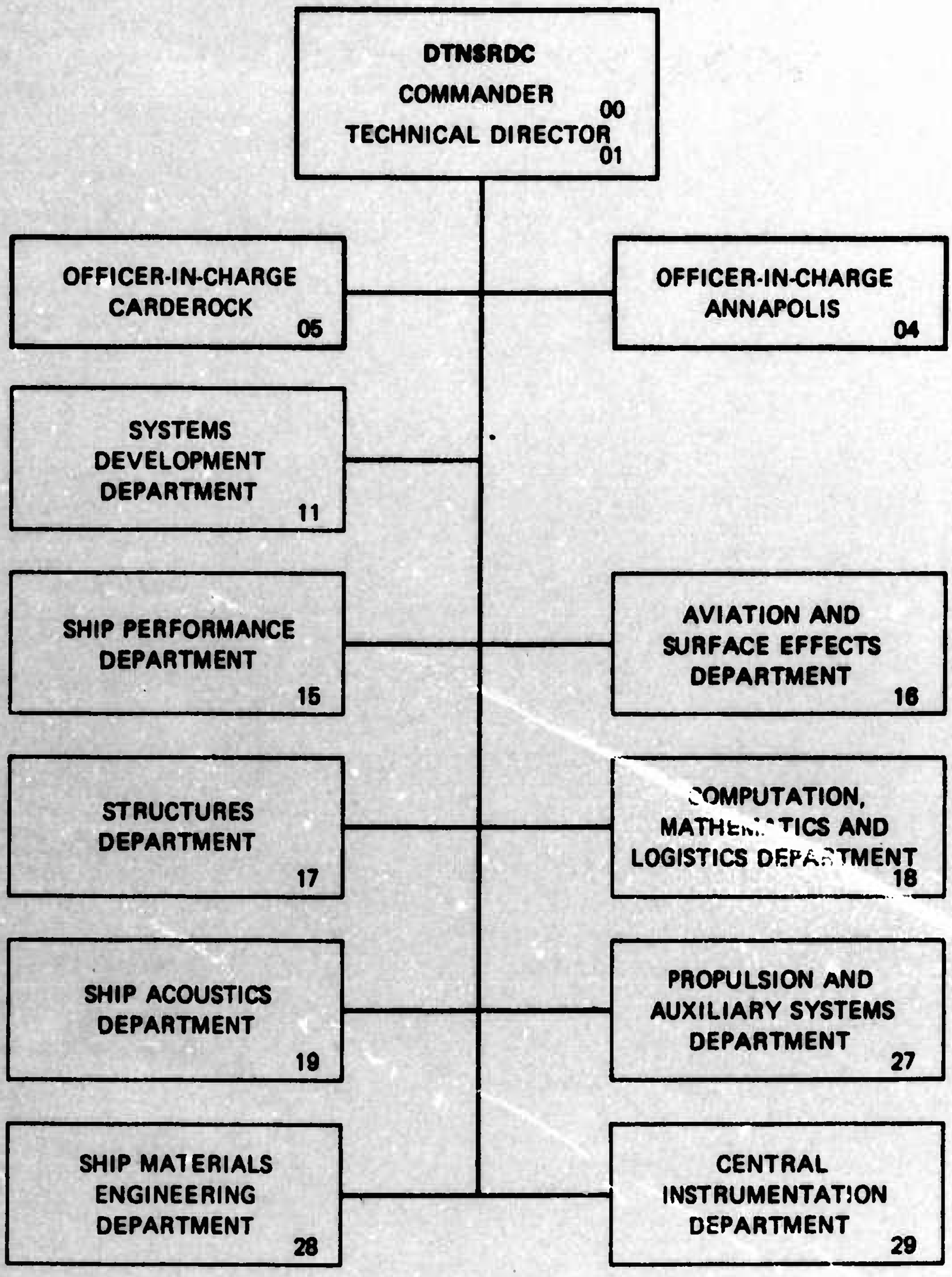

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NOMENCLATURE

The terminology is in general agreement with Reference 9. Unless otherwise noted, all of the following quantities are nondimensionalized in terms of one-half the fluid density and appropriate powers of the velocity of the origin and the body length. Following this table is a short summary of derivations for representative dimensionless forms. The numbers in brackets refer to applicable formulas in the text.

A, A*	Cross-sectional area of wake at stern
A_o	Effective area of body as an airfoil
A_t	Effective area of tail surface
a	Geometric aspect ratio of tail fin, including intersected portion of body
a_o	Effective aspect ratio of body alone
a_t	Effective aspect ratio of tail fin [22]
b	Actual semi-span of tail fin
b_o	Distance of tip of tail fin from body
b_t	Effective semi-span of tail fin [21]
c	Radius of gyration about y axis through center of gravity
C_{Dt}	Drag coefficient of tail fin, based on plan-form area of tail
C_{LO}	Lift coefficient of body alone, based on effective area of body
C_{Lt}	Lift coefficient of tail fin, based on plan-form area of tail
C_s	Surface area coefficient of body alone, based on circumferential area of circumscribing cylinder
C_t	Drag coefficient of body alone, based on surface area of body
D	Drag force
d	Diameter of body
F	Tail lift factor [12]
G	Downwash factor for rectilinear motion, [13]
H	Boundary-layer shape parameter, [68]
K	Downwash factor for rotary motion, [14]
I	Moment of inertia about y axis through center of gravity
k_1	Longitudinal virtual-mass coefficient

k_2	Transverse virtual-mass coefficient
k'	Virtual moment of inertia relative to y axis
L	Lift force, normal to direction of motion
L_w	Static lift derivative
l	Length of body (dimensional)
M	Moment about y axis through center of gravity
m	Mass of displaced fluid
M_q	Rotary-moment derivative, derivative of moment with respect to angular velocity
M_{qo}	Rotary-moment derivative for hull alone
M_{qt}	Rotary-moment derivative for tail fin
$M_{\dot{q}}$	Derivative of moment with respect to angular acceleration
M_w	Static-moment derivative, derivative of moment with respect to velocity in z direction
M_{wo}	Static-moment derivative for hull alone
$M_{\dot{w}}$	Derivative of moment with respect to acceleration in z direction
q	Angular velocity about center of gravity
R	Radius of hull at fin
r_o	Nose-radius coefficient
r_1	Tail-radius coefficient
U	Velocity of center of gravity relative to fluid (dimensional)
w	Velocity in z direction, normal velocity
$\bar{x}$	Absolute value of axial distance of center of buoyancy from bow
x_c	Absolute value of distance of point of application of lift force from center of gravity
x_s	Absolute value of distance of center of gravity from stern
x_t	Absolute value of distance from aerodynamic center of fin to center of gravity
Z	Normal force, positive downwards
Z_q	Rotary normal-force derivative
Z_{qo}	Rotary normal-force derivative for hull alone
$Z_{\dot{q}}$	Derivative of normal force with respect to angular acceleration

Z_w	Static normal-force derivative, derivative of force with respect to velocity in z direction
Z_{wo}	Static normal-force derivative for hull alone
Z_{wt}	Static normal derivative for tail fin
$Z_{\dot{w}}$	Derivative of normal force with respect to acceleration in z direction
α	Angle of attack in radians
β	Effective angle of attack on the tail fins
γ	Ratio of downwash angle at fin to downwash angle at center of body lift
Δ_1	A factor, [46]
Δ_2	A factor, [50]
δ^*	Displacement thickness of boundary layer at tail, [65]
ϵ	Angle of downwash, in radians
ζ_1	Ratio of lift coefficient according to Weinig's theory to lift coefficient according to lifting-line theory
ζ_2	Wake factor, correcting for velocity retardation over tail fin due to effect of hull boundary layer, [73]
θ	Momentum thickness of boundary layer at tail, [66]
λ	Length-diameter ratio of body
μ	An error factor, [48]
ζ_m	Axial distance of maximum section, body lengths from bow
ρ	Mass density of fluid (dimensional)
σ	Directional-stability index, a root of the characteristic equation of the linearized equations of motion

REPRESENTATIVE NONDIMENSIONAL FORMS

$$A^* = \frac{\text{displacement area of wake}}{l^2}$$

$$M = \frac{\text{moment}}{\frac{1}{2}\rho U^2 l^3}$$

$$M_q = \frac{\partial\,(\text{moment})}{\partial(\text{angular velocity})}\ \frac{1}{\frac{1}{2}\rho l^4 U}$$

$$M_w = \frac{\partial(\text{moment})}{\partial \alpha} \frac{1}{\frac{1}{2}\rho U^2 l^3}, \text{ for small } \alpha$$

$$M_{\dot{w}} = \frac{\partial(\text{moment})}{\partial(\text{acceleration in z-direction})} \frac{1}{\frac{1}{2}\rho l^4}$$

$$m = \frac{2(\text{volume})}{l^3}$$

$$q = (\text{angular velocity}) \left(\frac{l}{U}\right)$$

$$r_o = \frac{(\text{nose radius})\, l}{(\text{diameter of hull})^2}$$

$$Z = \frac{\text{normal force}}{\frac{1}{2}\rho U^2 l^2}$$

$$Z_q = \frac{\partial(\text{normal force})}{\partial(\text{angular velocity})} \frac{1}{\frac{1}{2}\rho l^3 U}$$

$$Z_w = \frac{\partial(\text{normal force})}{\partial \alpha} \frac{1}{\frac{1}{2}\rho l^2 U^2}, \text{ for small } \alpha$$

$$Z_{\dot{w}} = \frac{\partial(\text{normal force})}{\partial \dot{w}} \frac{1}{\frac{1}{2}\rho l^3}$$

ABSTRACT

Various methods of estimating the forces and moments on an elongated body of revolution at a small angle of attack in uniform motion or with a small angular velocity in rotary motion are considered. After a review and critique of the literature, a new method for estimating the stability derivatives is proposed, the principal novel feature being the incorporation of a downwash correction due to lift on the hull in determining the lift on a tail surface.

The values of the stability derivatives given by the various sets of formulas considered are compared with the results of measurement, both with a 3-component dynamometer and with an oscillator, on two bodies of revolution, with and without tail surfaces.

Two applications of the formulas for the stability derivatives are made. In one they are used to determine the size of tail surfaces necessary in order to obtain a prescribed value of the stability index; in the other they are applied to derive simple expressions and curves for the errors in the stability index due to percentage errors in the stability derivatives.

INTRODUCTION

In the era from about 1920 to 1935, the development of the airship stimulated considerable research in the aerodynamics of elongated bodies. In the main the results of this work have been thoroughly reviewed by Munk,[1] and Arnstein and Klemperer.[2] Unfortunately, as a consequence of the airship disasters of the 1930's, airship construction ceased and this stimulus for research was lost.

During the past war the design of numerous weapons and devices taxed the existing theory of the forces and moments on elongated bodies of revolution and clearly indicated the necessity for additional work in this field. The new ideas and procedures developed by the Germans in connection with their investigations of the stability and motions of submarines and torpedoes have been reported principally by Albring.[3,4,5] In this country a procedure for estimating the hydrodynamic characteristics of a body of revolution, equipped with tail fins, when its bare hull characteristics are known or assumed, was developed on the basis of Freeman's AKRON data[6] by one of the present authors and employed in the design of underwater bodies. The presentation of a refined version of this procedure is one of the purposes of this

[1]References are listed on page 35.

report.

In the postwar period experimental data on the hydrodynamic characteristics of elongated bodies of revolution have been accumulated at the Experimental Towing Tank of the Stevens Institute of Technology and at the Taylor Model Basin; concurrently the work of Munk[1] on the inviscid flow about such bodies has been extended by Laitone.[7] Also, within the last decade, significant advances have been made in boundary-layer theory, especially on the growth of the boundary layer along a body in the presence of a pressure gradient. These are reviewed in a recent report by Granville.[8]

On the whole, however, it appears that the principal aim of this research—to find a theory for the flow about an elongated body moving through a viscous fluid which is in accord with the observed flow and which predicts forces and moments in agreement with experiment—is far from attained. Nevertheless, several important principles have been established on the basis of which it appears possible to make rational estimates of these forces and moments.

It is proposed in the present report to review some of these principles and procedures, to derive the aforementioned new procedure, and conversely to show how it may be applied to determine the size of appendages necessary to obtain a desired value for the dynamic-stability index. As an interesting application of one of the simpler sets of prediction formulas, an appendix on the sensitivity of the value of the stability index to errors in the values of the stability derivatives is also included.

THE FRAME OF REFERENCE

The longitudinal, transverse and normal axes x, y, and z, respectively, are assumed to rotate with the body, with the origin of the coordinate axes at the center of gravity. Positive directions are x forward, y to starboard and z downward, perpendicular to x and y. The respective forces X, Y, and Z and velocities u, v, and w are directed accordingly. For the purpose of this analysis motion will be assumed confined to the xz plane, with angle of attack α, angular velocity q and moment M about the y axis assumed positive if directing the positive z axis to rotate into the direction of the positive x axis.

The foregoing conventions are in accordance with the nomenclature of SNAME Bulletin 1-5,[9] which will also be employed in the following.

THE CHARACTERISTIC EQUATION; THE STABILITY DERIVATIVES

Neglecting metacentric stability and surface effects, the characteristic equation of the linearized differential equations of motion in the vertical plane, in dimensionless form, may be written as[10]

$$[(Z_{\dot{w}}-m)(M_{\dot{q}}-I) - M_{\dot{w}}Z_{\dot{q}}]\sigma^2 + [Z_w(M_{\dot{q}}-I) + M_q(Z_{\dot{w}}-m) - Z_{\dot{q}}M_w - M_{\dot{w}}(m+Z_q)]\sigma$$

$$+ [Z_wM_q - M_w(m+Z_q)] = 0 \qquad [1]$$

In this equation the primes usually employed to distinguish between dimensional and nondimensional quantities[9] have been omitted, since all quantities are dimensionless. In [1], m is the mass of the body, I its moment of inertia about the y-axis through the center of gravity; the dot over a variable denotes the time derivative of the variable, and a variable appearing as a subscript to Z or M denotes partial differentiation with respect to that subscript. Quantities are nondimensionalized by dividing by appropriate powers of the basic quantities l, U, and $\frac{1}{2}\rho$, where l is the length of the body, U is its speed of advance, and ρ is the mass density of the fluid.

The characteristic equation is a quadratic from which the stability indices σ may be determined when the other quantities are known. It will be shown that it may also be used to estimate the size of the stabilizing surfaces when a desired value of a stability index is prescribed.

When numerical values are considered, many of the terms in [1] are found to be small in comparison with others. It is customary to assume approximate simplifying values for these small terms. For a nearly neutrally-buoyant body these assumptions are the following:

$$Z_{\dot{w}} = -k_2m, \quad Z_{\dot{q}} = 0, \quad M_{\dot{w}} = 0, \quad M_{\dot{q}} = -k'I \qquad [2]$$

where k_2 and k' are coefficients of additional lateral mass and of moment of inertia about a transverse axis.[1] The determination of the remaining stability derivatives, Z_w, M_w, Z_q, M_q, is the objective of various experimental techniques and theoretical analyses. Some of the methods for estimating these derivatives will now be discussed.

REVIEW OF THEORY FOR STABILITY DERIVATIVES FOR A BODY OF REVOLUTION WITHOUT APPENDAGES

LAMB'S ANALYSIS

It is instructive first to examine the values of these derivatives obtained from potential-flow theory. From Chapter VI of Lamb's Hydrodynamics[11] the following values may be derived

$$\begin{aligned}
&(a)\quad Z_w = 0 \\
&(b)\quad M_w = (k_2 - k_1)m \\
&(c)\quad Z_q = k_1 m \\
&(d)\quad M_q = 0
\end{aligned} \qquad [3]$$

where k_1 is the coefficient of additional axial mass.[1]

The first of Equations [3] is known to be in serious error. Concerning this Arnstein and Klemperer[2] remark: "When an airship is propelled at an angle of attack, lift forces are created in a similar manner as by the wing of an airplane. It is true that the airship's shape as a wing is very poor and its aspect ratio extremely small; but the size of the exposed surfaces is so great that tremendous aerodynamic force components at right angles to the flight path can be evoked." This discrepancy is attributable to viscous effects which contribute—to the forces and moments acting on the body—not only the integrated effects of the shearing stresses, but also, by the formation of a boundary layer along the body and a wake, greatly modify the pressure distribution, especially toward the after part of the body. This discrepancy between the potential-flow pressure distributions and the measured values on a model of the USS AKRON is graphically shown by Allen.[12]

The values for M_w in [3] (known as Munk's formula), are generally about 15 percent higher than the measured values. This discrepancy is also attributable to the viscosity of the fluid which, by diminishing the downward force acting near the stern, decreases the moment and gives a resultant upward force.

The value for Z_q in [3] corresponds to an outward (centrifugal) force exerted by the fluid on a body in rotary motion. This force is due to the uniform rate of change of direction of the longitudinal momentum imparted to the fluid by a body moving in a circular path. Since it is known that virtual-mass effects are only slightly influenced by viscosity, it is reasonable to assume that the term $k_1 m$ will contribute to the value of Z_q in addition to any effects of viscosity.

The little available data for M_q indicates that the last of Equations [3] is very nearly correct for a viscous fluid also. The contribution of viscosity to this and the other derivatives will be discussed more fully in a subsequent section.

LAITONE'S ANALYSIS

An interesting modification of the potential-flow theory, in which it was attempted to take into account the effect of viscosity by assuming that the body effectively had an area of section at the stern equal to the width of its wake, has been carried through by Laitone.[7] On the assumption that the center of gravity and buoyancy coincide, his results may be expressed as follows:

$$
\begin{aligned}
&(a)\quad Z_w = -2A,\\
&(b)\quad M_w = m + x_s Z_w,\\
&(c)\quad Z_q = k_1 m + x_s Z_w,\\
&(d)\quad M_q = x_s^2 Z_w
\end{aligned}
\qquad [4]
$$

where A is the area of section of the wake at the stern, nondimensionalized in terms of the body length, and x_s is the distance of the center of gravity from the stern similarly nondimensionalized. Since the diameter of the wake is unknown, [4] does not give an estimate for Z_w, but valid expressions for the other derivatives in terms of Z_w would be most useful.

Laitone's formulas are based on a section-element theory which assumes that at each section the fluid has the lateral momentum corresponding to the two-dimensional virtual mass of the section. No account is taken of the lift and downwash effects of the vortex system which Harrington,[13] Engelhardt,[14] and Albring[5] have shown to exist in the flow field of a yawed body. Results from the formulas are compared with experiment in a subsequent section.

ALBRING'S ANALYSIS

In References 3 and 4 Albring has derived formulas for the stability derivatives on the basis of the assumption that the lift force may be considered to act at the same point near the stern on the axis of the body for both straight and rotary motion, whether or not the body is equipped with fins. Furthermore, he assumes, in Reference 3, that the lift force on the body in rotary motion is equal to that for motion in a straight line at an angle of attack equal to the local angle of attack in rotary motion at the point of application of the lift force. In Reference 4 the latter assumption is modified by reducing the force thus obtained by a constant amount so as to obtain zero force on a hull due to rotary motion.

Albring's results, applied to a hull without fins, are as follows:

$$\begin{aligned}
&(a)\quad L_w \text{ assumed or measured,}\\
&(b)\quad M_w = (k_2 - k_1)m - x_o L_w,\\
&(c_1)\quad Z_q = -x_o L_w \text{ (from Reference 3),} \qquad [5]
\end{aligned}$$

or

$$\begin{aligned}
&(c_2)\quad Z_q = 0 \text{ (according to Reference 4),}\\
&(d)\quad M_q = -x_o^2 L_w
\end{aligned}$$

where L is the nondimensionalized force normal to the direction of incident flow and x_o is the nondimensionalized distance of the assumed point of application of the lift force from the center of gravity. Z_w is related to L_w by the equation

$$Z_w = -(L_w + D) \qquad [6]$$

where D is the nondimensionalized drag of the body. It is implied by Albring that x_o is approximately half the prismatic coefficient of the body.

Since Reference 4 was written after 3, it must be presumed that Albring preferred the value of Z_q in [5c_2] derived from Reference 4. The values of the derivatives given by the various formulas will be compared with experiment in a subsequent section.

REVIEW OF THEORY OF STABILITY DERIVATIVES FOR A BODY OF REVOLUTION WITH TAIL SURFACES

POTENTIAL FLOW FORMULAS

In estimating the forces and moments on an elongated body of revolution equipped with tail surfaces it has been customary to assume that there is no interference between the body and these surfaces, so that their separate effects are additive. Let Z_{wt} be the contribution of the tail surfaces to Z_w and let x_t be the distance (nondimensionalized) between the center of gravity and the center of pressure on the tail. Then, the addition of the tail-surface effects to the values for the body alone gives

$$\begin{aligned}
&(a)\quad Z_w = Z_{wo} + Z_{wt}\\
&(b)\quad M_w = M_{wo} + x_t Z_{wt}\\
&(c)\quad Z_q = Z_{qo} + x_t Z_{wt} \qquad [7]\\
&(d)\quad M_q = M_{qo} + x_t^2 Z_{wt}
\end{aligned}$$

Here Z_{wt} is to be estimated from the airfoil theory of low-aspect-ratio airfoils, and the zero subscript denotes values (such as Equations [4] and [5]) for the body of revolution without tail surfaces.

The terms $x_t Z_w$ and $x_t^2 Z_w$ in Equations [7c] and [7d] are obtained by the well-known procedure of computing the force on a tail surface, when the (nondimensionalized) angular velocity of the body is q, from the mean lateral velocity of the surface, $w = qx_t$. This gives $Z_{wt} x_t q$ and $Z_{wt} x_t^2 q$ for the (nondimensionalized) force on the surface and the moment of this force about the center of gravity. The derivative of this force and moment with respect to q then gives the terms in Equations [7c] and [7d].

Worthy of special note are the extremely simple approximations obtained for an elongated body by setting $k_1 = 0$, $k_2 = 1$, and $x_t = \frac{1}{2}$ in [3] and substituting into [7]:

$$\begin{aligned} &(a) \quad Z_w = Z_{wt} \\ &(b) \quad M_w = m + \tfrac{1}{2} Z_w \\ &(c) \quad Z_q = \tfrac{1}{2} Z_w \\ &(d) \quad M_q = \tfrac{1}{4} Z_w \end{aligned} \qquad [8]$$

ALBRING'S ANALYSIS

As was stated in the discussion of the body without fins, Albring assumed that the point of application of the lift force was unaltered by the addition of fins. Here also there are two formulas for Z_q, one obtained from Reference 3, and a "corrected" one obtained from Reference 4. His results for a hull with fins may be expressed as follows:

$$\begin{aligned} &(a) \quad L_w \text{ assumed or measured,} \\ &(b) \quad M_w = (k_2 - k_1)m - x_o L_w, \\ &(c_1) \quad Z_q = - x_o L_w \text{ (from Reference 3),} \\ \text{or} \quad &(c_2) \quad Z_q = - x_o (L_w - L_{wo}) \text{ (from Reference 4)} \\ &(d) \quad M_q = - x_o^2 L_w \end{aligned} \qquad [9]$$

Here L_{wo} denotes the lift rate for the bare hull as distinguished from L_w, the lift rate for the hull with fins. x_o is the nondimensionalized distance of the assumed point of application of the lift force from the center of gravity, implied by Albring to be half the prismatic coefficient of the body.

Equation [6], $Z_w = -(L_w+D)$, is still applicable, although the terms now refer to the body with fins.

Results from Equations [9] will be compared with experiment in a subsequent section.

DERIVATION OF NEW FORMULAS FOR STABILITY DERIVATIVES

Several investigators[5,13,14] have observed that at moderate angles of attack there is a vortex system in the wake of an elongated body of revolution analogous to that of an airfoil of low aspect ratio. For the purpose of developing an approximate theory it will be assumed that, for small angles also, a part of the lift developed by the body is attributable to its action as a symmetrical airfoil of low aspect ratio.

At small angles of attack α, the lift of a body of revolution is expressible in the form $L = C_1\alpha + C_2\alpha^2$. It is well known that a very long cylinder at an angle of attack is subject to a normal force proportional to $\sin^2\alpha$, (Reference 15), an effect which may be explained by assuming that the longitudinal and normal components of the incident flow act independently. It appears reasonable to suppose that this effect accounts for the quadratic term at small angles of attack. However, since in the present work we are concerned only with the estimation of the stability derivatives, to which the quadratic term makes no contribution, we will not consider this term any further.

The original assumption can now be stated more precisely, that, at small angles, that part of the lift which varies linearly with the angle of attack may be treated by the methods of airfoil theory.

According to the theory of the lifting line,[16] as modified by Weinig's cascade theory,[17] at each point downstream from the hull center of lift there is a downwash angle ϵ, a function of position, determined by the lift coefficient and the effective aspect ratio of the hull. Thus the resultant angle of attack at a point is

$$\beta = \alpha - \epsilon \qquad [10]$$

The analytical determination of the load distribution and lift on a tail surface is now seen to be a very difficult problem. Not only is the tail immersed in a thick boundary layer and subject to its velocity gradients, but also the proximity of the tail to the hull center of lift,[13,18] implies variations of the downwash angle along the chord and span of the tail surface. Nevertheless, an approximate analysis will be carried through by assuming mean values of the downwash angle and flow velocity over the tail surfaces.

The following approximate formulas will be derived:

$$\begin{aligned}
&(a)\quad Z_w = Z_{wo} - FA_tG \\
&(b)\quad M_w = M_{wo} - x_tFA_tG \\
&(c)\quad Z_q = Z_{qo} - FA_tK \\
&(d)\quad M_q = M_{qo} - x_tFA_tK
\end{aligned} \qquad [11]$$

Here F, G, and K denote the expressions

$$F = \frac{2\pi\zeta_1\zeta_2}{1 + 2/a_t} \qquad [12]$$

$$G = 1 - \frac{1}{\pi}\gamma\lambda^2 L_{wo} \qquad [13]$$

$$K = x_t - \frac{0.10}{\pi}\gamma\lambda^2 m \qquad [14]$$

where A_t is the effective area of the tail surfaces,

x_t is the distance (nondimensionalized) between the center of gravity and the center of pressure on the tail,

ζ_1 is Weinig's correction factor, referred to as ζ_a in Reference 21, for small aspect ratio

ζ_2 is the wake factor, correcting for the difference between the mean dynamic pressure at the tail surface and the dynamic pressure in free stream,

γ is the ratio of the mean downwash angle at the tail to the downwash angle at the center of body lift,

λ is the length-diameter ratio for the body, and

a_t is the effective aspect ratio of tail fin.

The bare-hull derivatives Z_{wo}, M_{wo}, Z_{qo}, M_{qo} can either be taken from experiment or from Laitone's or Albring's formulas, [4] and [5]. This will be discussed in the following sections in which the Equations [11] are derived.

EFFECTIVE ANGLE OF ATTACK AT THE TAIL

Weinig's theory[17] shows that for small angles of attack the downwash angle for a surface of small aspect ratio also approaches the value given by lifting-line theory

$$\epsilon = \frac{\gamma C_{LO}}{\pi a_o} \qquad [15]$$

where C_{LO} is the lift coefficient for the lift on the body based on its effective area as an airfoil,

a_o is the effective aspect ratio of the body, and

γ is the ratio of the mean angle of downwash at the tail to the angle at the center of lift.

If A_o is the effective area of the body as an airfoil nondimensionalized in terms of the body length, we have $C_{LO} = L_{WO}\,\alpha/A_o$. Furthermore, by definition, $a_o = 1/A_o\lambda^2$. Substituting these values into [15] gives

$$\epsilon = \gamma\lambda^2 L_{WO}\,\alpha/\pi \qquad [16]$$

Hence, from [10] and [16] we obtain

$$\beta = \alpha G \qquad [17]$$

where

$$G = 1 - \frac{1}{\pi}\gamma\lambda^2 L_{WO} \qquad [18]$$

The ratio γ at first increases with distance downstream from a value of unity at the center of lift, reaches a maximum, and then decreases asymptotically. Experimental and theoretical investigations are needed to define the variation of γ with body form and position relative to the body.

FORCE ON A TAIL SURFACE

According to lifting-line theory, assuming elliptic load distribution, the lift coefficient for an isolated wing surface of aspect ratio a at an angle of attack β is

$$C_L = \frac{2\pi\beta}{1 + 2/a} \qquad [19]$$

For small aspect ratios, this equation leads to results which are considerably in error. The application of Weinig's[17] correction factor ζ_a, herein referred to as ζ_1, to Equation [19] gives improved agreement with experiment.[18] For $\alpha = 0$, Weinig's formula becomes

$$\zeta_1 = \frac{(1 + a/2)\tanh(2/a)}{1 + \tanh(2/a)} \qquad [20]$$

A graph of ζ_1 against aspect ratio is given in Figure 1.

If such a wing is attached to a hull, however, measurements show that the loading is somewhat reduced near the hull but that additional lift is induced on the portions of the hull adjoining the wing. Furthermore, because at least part of the surface will be within the boundary layer of the hull, there will occur a nonuniform reduction in the velocity over the surface.

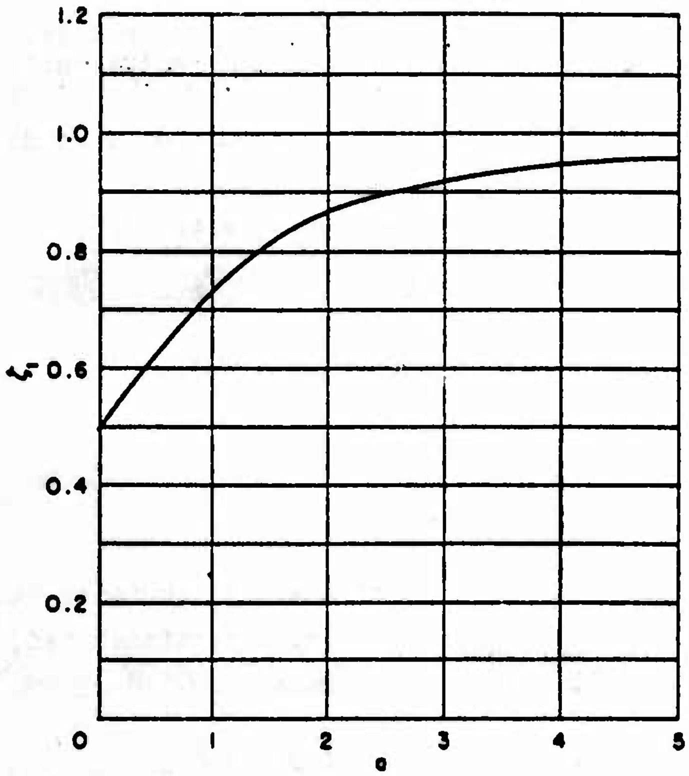

Figure 1 - Limit of ζ_1 (Ratio of Lift Coefficient According to Weinig Theory to Lift Coefficient According to Lifting Line Theory) as Angle of Attack Approaches Zero, Plotted as a Function of Aspect Ratio

It will be assumed that the effect of the boundary-layer flow over the tail surface can be expressed by the application of another correction factor ζ_2 to [19], to account for a mean reduction in incident flow. The deviation from elliptic load distribution, due both to the transverse velocity gradients in the boundary layer and to the image effects in the hull, may also modify Equation [19], but it is not considered worthwhile to attempt to correct for this effect in the present approximate theory.

The additional lift on a hull adjacent to a lifting surface is discussed by Wieselsberger.[20] His results have been applied by Fehlner[21] to obtain approximate formulas for the effective span and effective aspect ratio of an isolated wing having the same lift and induced drag as the combination of intersecting wing and hull. The formulas are

$$b_t = b\left(1 - \frac{R^2}{b^2}\right) \qquad [21]$$

$$a_t = (a + 2)\left[1 - \frac{R^2}{b^2}\right] - 2 \qquad [22]$$

where b and b_t are the actual (wing tip to body axis) and effective semi-spans,

R is the hull radius, which Multhopp[22] recommended be taken at the 3/4 chord point when the radius varies along the root of the wing, and

a and a_t are the actual (including intersected area of body) and effective aspect ratios.

The effective area of the wing is then given by $A_t = 4b_t^2/a_t$.

With the aforementioned corrections, [19] becomes

$$C_{Lt} = F\beta \qquad [23]$$

where

$$F = \frac{2\pi\zeta_1\zeta_2}{1 + 2/a_t} \qquad [24]$$

Let C_{Dt} be the drag coefficient of the tail surface, based on its projected area. Then, for small angles, the (nondimensionalized) normal component of the force on the tail may be written as

$$Z_t = -A_t(C_{Lt} + \beta C_{Dt}) = -A_t\beta(F + C_{Dt}) \qquad [25]$$

from [23]. In general, C_{Dt} is negligible in comparison with F. Hence, substituting for β from [17] and differentiating [25] with respect to α gives

$$Z_{wt} = -FA_tG \qquad [26]$$

since, by symmetry,

$$\frac{dF}{d\alpha} = \frac{dG}{d\alpha} = 0$$

when $\alpha = 0$.

THE STATIC NORMAL-FORCE DERIVATIVE, Z_w

The superposition of the hull and tail-surface contributions to Z_w immediately gives Equation [11a].

It should be noted that a theoretical method of estimating Z_{wo} is not yet available. Laitone's formula, [4], $Z_{wo} = -2A$, only expresses Z_{wo} in terms of another unknown. Hence, it is recommended that Johnson's empirical formula,[18] based on tests of a large series of bodies, be used:

$$Z_{wo} = -(0.234\ m^{0.79} + D) \qquad [27]$$

THE STATIC-MOMENT DERIVATIVE, M_w

The moment about the center of gravity of the force on the tail surface is $M_t = x_t Z_t$, and hence, from [26]

$$M_{wt} = x_t F A_t G \qquad [28]$$

For the complete assembly, then

$$M_w = M_{wo} - x_t F A_t G$$

in accordance with Equation [11b].

Here M_{wo} may be estimated from Laitone's Equation [4b] or from Johnson's empirical formula[18]

$$M_{wo} = 0.87\ (k_2 - k_1)m \qquad [29]$$

THE ROTARY NORMAL-FORCE DERIVATIVE, Z_q

Consider a body of revolution, without tail surfaces, moving in a circular path at a steady nondimensional angular velocity, q, with zero angle of attack at the center of gravity. It was seen, in the discussion following Equation [3], that the inertia of the fluid contributes a term $k_1 m$ to the value of Z_q. Also, by the analogy between a straight body in curved flow and a curved model in straight flow, it might be expected that the body would experience a lift force, like a cambered airfoil at zero angle of attack. In the following section, after a consideration of Gourjienko's measurements on a curved model of an airship hull[23] and rotating arm measurements at the Stevens Institute of Technology on a torpedo model without fins,[24] the mean value -0.10 m is taken as the contribution due to this effect. Thus the rotary derivative for the bare hull will be taken as

$$Z_{qo} = -\ (0.10 - k_1)m \qquad [30]$$

It will be supposed that the part of this force additional to the inertia effect is of the same nature as that acting on a straight body at an angle of attack in uniform flow and hence, the effects of the resulting downwash will be included in the calculation of the tail-surface force due to rotation.

The contribution of the tail surfaces to Z_q will now be considered. The angular velocity q imparts a mean normal velocity $w = qx_t$ to the surface; or, since all quantities are nondimensionalized, the angle of attack on the tail surface due to the angular velocity is $\alpha = qx_t$. Also, corresponding to the normal force 0.10 mq, there is a downwash angle at the tail surface, analogous to [16],

$$\epsilon = \frac{0.10}{\pi} \gamma \lambda^2 mq \qquad [31]$$

Hence, the effective angle of attack on the tail surface is

$$\beta = qK \qquad [32]$$

where

$$K = x_t - \frac{0.10}{\pi} \gamma \lambda^2 m \qquad [33]$$

Then, from [25], again neglecting the drag of the tail surface,

$$Z_t = \beta F A_t = - qFA_t K$$

and hence,

$$Z_{qt} = - FA_t K \qquad [34]$$

since, by symmetry,

$$\frac{dF}{d\alpha} = \frac{dK}{d\alpha} = 0$$

when $\alpha = 0$.

The resultant value for the body with tail surfaces is now seen to be in accordance with [11c].

THE ROTARY-MOMENT DERIVATIVE, M_q

The moment about the center of gravity of the force on the tail surface is $M_t = x_t Z_t$ and hence, from [34],

$$M_{qt} = - x_t FA_t K \qquad [35]$$

For the complete assembly, then

$$M_q = M_{qo} - x_t FA_t K$$

in accordance with [11d].

COMPARISON WITH EXPERIMENT

In order to evaluate the procedures indicated in the preceding text, two bodies of revolution of TMB Series 58 were subjected to further tests to determine the dynamic stability characteristics. This section describes these tests and compares the values thus obtained with values predicted by the various formulas.

DESCRIPTION OF MODELS, APPARATUS, AND TEST PROCEDURE

Series 58 Models 4164 and 4166 (References 18, 25 and 26) are 9-foot models of laminated mahogany. They were cut off a short distance forward of the stern and a reinforced white-metal tail-cone—stabilizer assembly was substituted. These models are shown in Figure 2. They were fitted so that they could be mounted at the center of buoyancy for static-force and moment measurements or at points 6 3/16 in. ahead or 7 3/16 in. aft of the center of buoyancy for testing on the underwater-body oscillator. Geometrical characteristics of the models are summarized in the following table.

TABLE 1

Characteristics of Models 4164 and 4166

Model	λ	ζ_m	C_p	r_o	r_1	$\bar{x}$	m	C_s	A_t	a_t	x_t
4164	7	0.40	0.55	0.50	0.10	0.430	.0176	.695	.00414	4.93	.564
4166	7	0.40	0.70	0.50	0.10	0.478	.0224	.810	.00602	3.39	.513

In Table 1 λ is the length-diameter ratio of the body,

ζ_m is the axial distance of the maximum section, in body lengths from bow,

C_p is the prismatic coefficient,

r_o is the nose-radius coefficient,[26]

r_1 is the tail-radius coefficient,[26]

$\bar{x}$ is the axial distance of the center of buoyancy, in body lengths from bow,

m is the nondimensional mass of the displaced fluid,

C_s is the surface-area coefficient, defined as the ratio of the surface area of the body to circumferential area of a circumscribing cylinder of the same length,

A_t is the effective plan-form area of the tail fin, nondimensionalized in terms of the body length,

a_t is the effective aspect ratio of the tail fin, and

x_t is the distance (nondimensionalized) between the center of gravity and the center of pressure on the tail.

The static-stability determinations[18] were made by means of a hydraulically operated three-component dynamometer, shown schematically in Figure 3, at speeds ranging from 3 to 12 knots for angle-of-attack settings of 0°, ±1°, ±2°, ±3°, and ±4°.

Rotary derivatives were determined by analysis of forced oscillations using the underwater-body oscillator[10] shown in Figure 4. This instrument produces torsional vibrations about a vertical axis. It consists of a driving or forcing mechanism, spring-coupled to a driven or following mechanism by means of a torsion bar, and a displacement-indicating system. Sinusoidal displacement is introduced at the head of the oscillator by means of a scotch yoke of variable stroke. A rheostat on the driving motor permits variation of the forcing frequency. Oscillator tests are made at two mounting positions, one usually forward of the center of buoyancy, and the other aft. For tests with tail fin, the resonant frequency and magnification were determined in runs at varied input frequency and amplitude for speeds ranging from 2 to 5 knots. In order to obviate running at resonance in the presence of the small damping occurring with the bare hull, the tests without fin were performed by means of the phase-lag method. The latter tests, performed on Model 4164 only, were executed at a constant speed of 6 knots under varied conditions of input amplitude and frequency.

EXPERIMENTAL RESULTS

The analysis of resonance data obtained with the oscillator is described in TMB Report C-124.[10] The phase-lag method will be described in a subsequent report. Analysis of the data gives M_q = .0002 for Model 4164. The data were not sufficiently precise to enable Z_q to be determined for the "without fins" condition. Even M_q for this condition is presented with great reservation, due to the suspected but yet unevaluated critical influence of

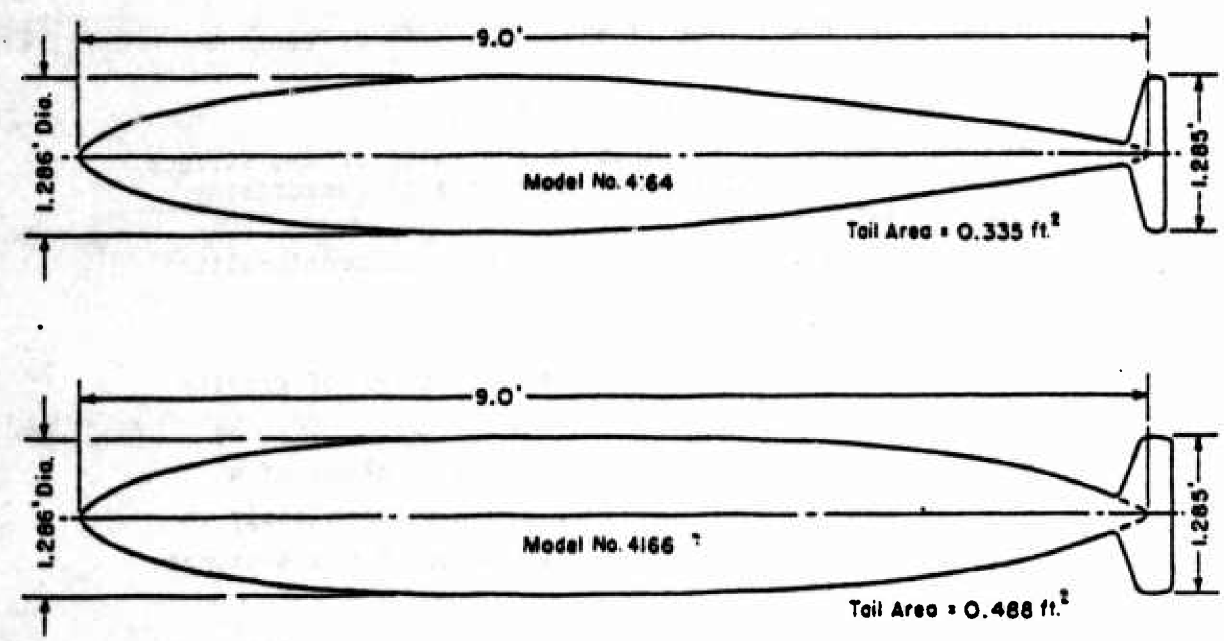

Figure 2 - Models Used in Experimental Investigation

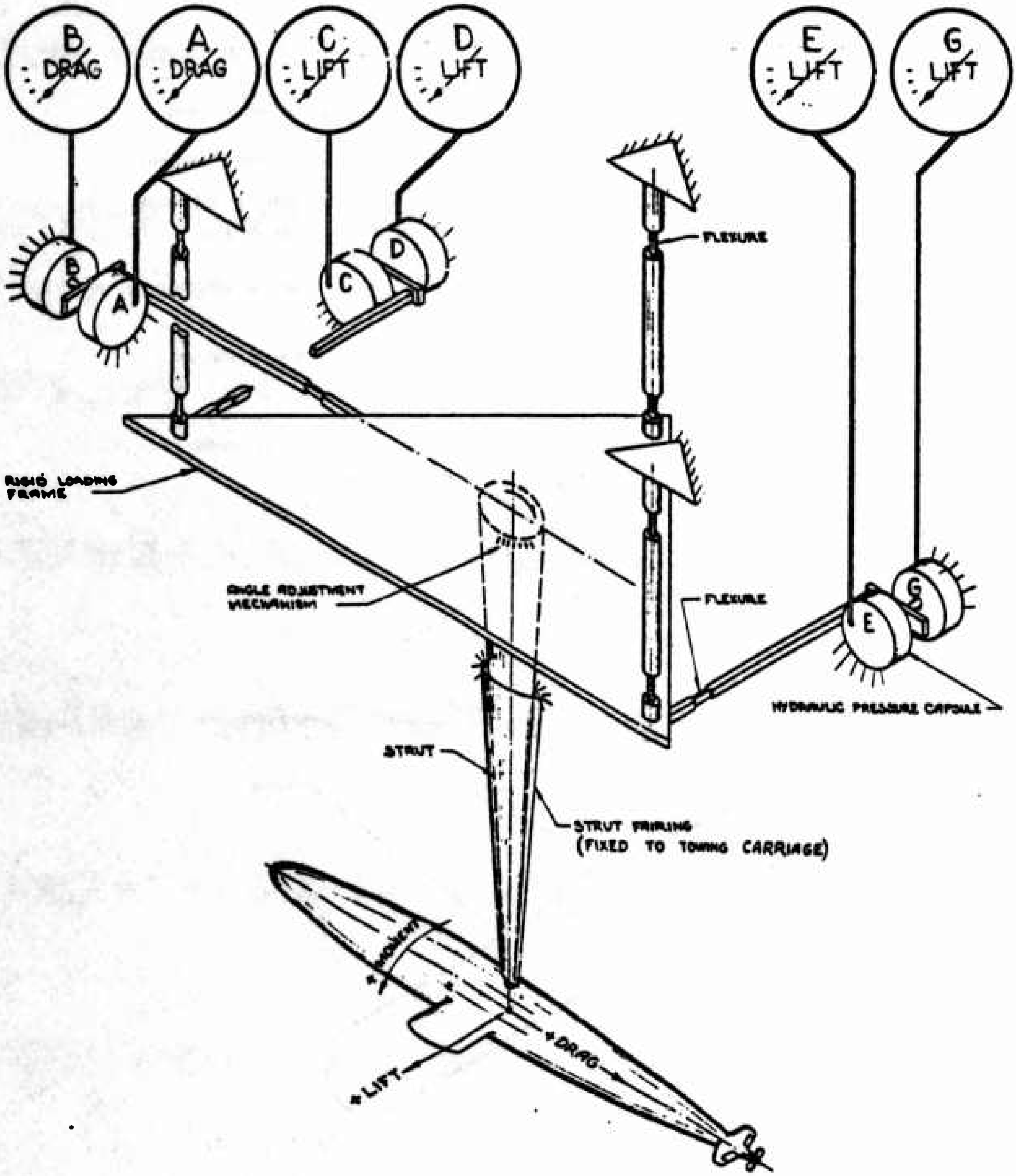

Figure 3 - Schematic of Three-Component Balance

unsteady lift effects upon the forces on an oscillating body. Indeed, it is believed that the value of M_q obtained in this test, in which the model travelled about three lengths during one complete oscillation, is much closer to the limiting value for infinite frequency (infinite Strouhal number) than to the steady (zero frequency) value. The steady value may be greater than the value obtained by a factor of 2 or 4.[27] This effect should be considerably less for a body equipped with fins since the chord lengths travelled

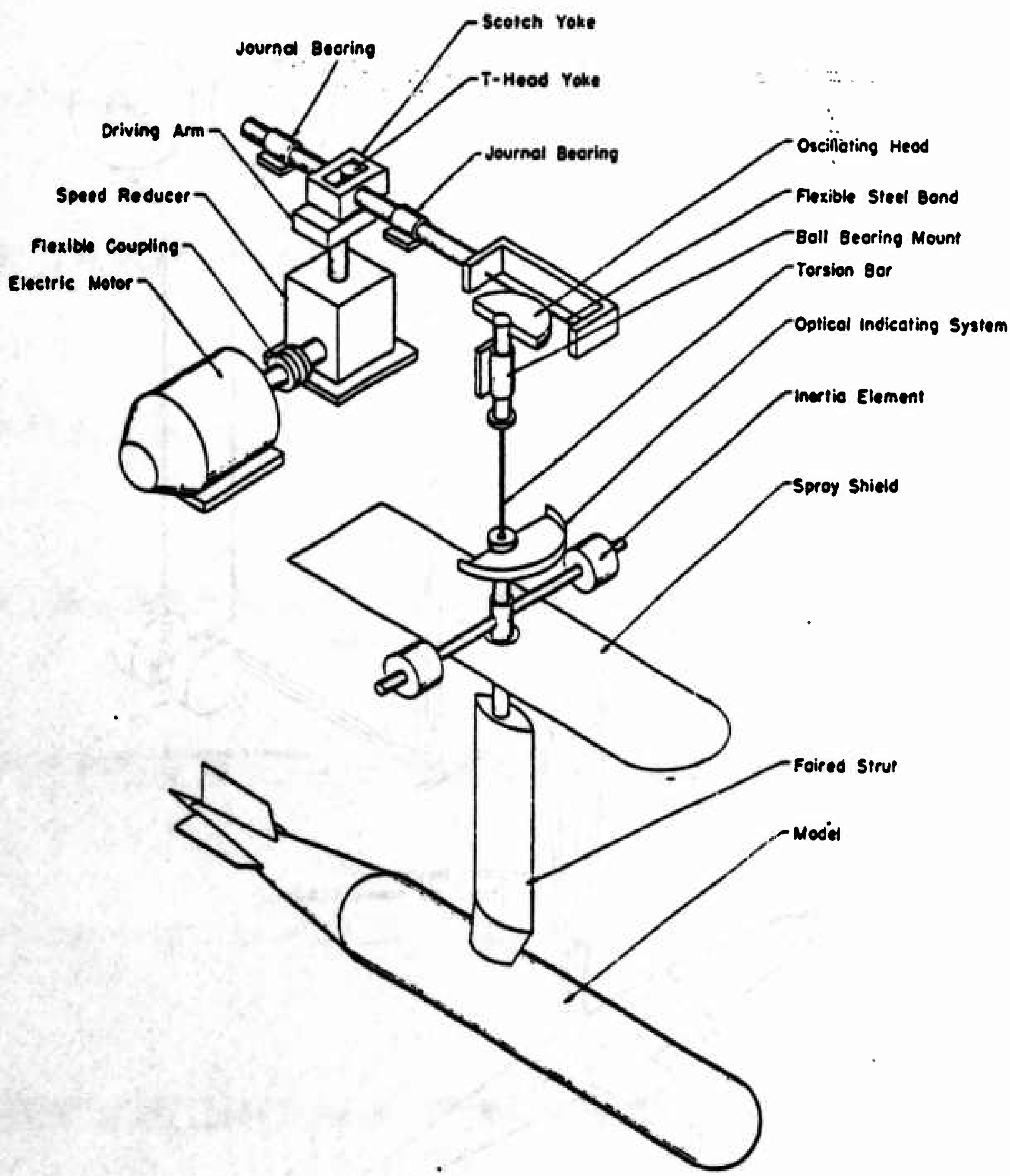

Figure 4 - Schematic Diagram of Underwater-Body Oscillator

per cycle is about 30 times as great for the fins as for the body, and the damping of rotary motion is principally due to the fins.

Rotary derivatives for an elongated body without fins have been measured by Gourjienko[23] on a curved airship model. His data indicate that for such a body

$$Z_q = -(0.08 - k_1)\, m \qquad [36]$$

where the first term is taken from Gourjienko's measurements in straight flow and the second is added as the contribution of the centrifugal reaction of the fluid for rotary motion. His measurements also give

$$M_q = -0.040 \, m \qquad [37]$$

In contrast with the above formulas from curved model tests, tests of a bare torpedo hull in the rotating-arm tank at the Stevens Institute of Technology[24] gave

$$Z_q = -(0.133 - k_1) \, m \qquad [38]$$

$$M_q = -0.053 \, m \qquad [39]$$

Both sets of values for M_q and Z_q must be considered uncertain, the former because of the fundamental assumption of the equivalence of curved models in straight flow to straight models in curved flow, the latter because of the difficulty of measuring the small lateral force on a bare hull in the presence of the large inertial centrifugal force on the model due to its rotary motion. Consequently, in the following comparison we will assume the mean values

$$Z_q = -(0.10 - k_1) \, m \qquad [40]$$

$$M_q = -0.045 \, m \qquad [41]$$

The results obtained from the measurements on Models 4164 and 4166 are given in Table 2. It is interesting to note that the value of M_q for Model 4164 without fins is about four times the value obtained from the oscillator experiment. This result is not inconsistent with the previous discussion in which it was anticipated that because of the unsteady motion the value of M_q obtained from the oscillator might be low by a factor of this magnitude.

TABLE 2

Experimental Values of Stability Derivatives

	Model 4164		Model 4166	
	With Fins	Without Fins	With Fins	Without Fins
Z_w	-.0200	-.0101	-.0210	-.0093
M_w	.0079	.0132	.0117	.0178
Z_q	-.0106	-.0011*	-.0098	-.0014*
M_q	-.0053	-.0008*	-.0053	-.0010*

*These values computed from [40] and [41] with $k_1 = 0.036$ for a body of length-diameter ratio $\lambda = 7$.

COMPARISON OF BARE HULL FORMULAS WITH EXPERIMENTAL VALUES

In order to apply Laitone's formulas, [4], it is necessary either to assume an empirical value for Z_w or to compute it from the cross-sectional area of the wake at the after end of the body. Both methods will be tried.

An expression for the area of the displacement wake of a body of revolution at zero angle of attack in terms of its drag, given by Granville,[27] is

$$A^* = \frac{C_s(H + 1)}{4\lambda} C_t \qquad [42]$$

where A^* is the displacement wake area (determined from the displacement thickness of the boundary layer) at the after end of the body, nondimensionalized in terms of the body length,

H is a boundary-layer shape parameter, and

C_t is the drag coefficient of the body alone, based on the surface area of the body.

Formula [42] has also been confirmed by application to the AKRON data.[6] A good approximation for H at the tail, suitable for streamlined bodies of revolution, given in References 28 and 29, is $H = 1.6$. An average value for C_t for the Reynolds number range of the model tests (about 10^{+7}, based on the length of the model) is $C_t = 0.003$. Hence, using the values of C_s and λ given in Table 1, we obtain:

Model	A^*	Z_w	
		Laitone (Theor.)	Johnson (Exp.)
4164	.000608	-.00122	-.0101
4166	.000709	-.00142	-.0093

It is seen that Laitone's theoretical formula for Z_w for a bare hull, which has also been given by Allen,[30] fails to give even the proper order of magnitude. A successful theory for the lift on an elongated body of revolution must consider the vortex system that has been observed in the wake of these bodies; this was not done by either Laitone[7] or Allen.[30]

When experimental values of Z_w are assumed, values of the other derivatives can be computed from Laitone's formulas, [4], and from Albring's formulas, [5]. The results are compared with the experimental values for the body without fins in Table 3.

Comparing the experimental values with the results computed by Laitone's and Albring's formulas, we see that either formula gives values for M_w agreeing fairly well with the experimental value, while for the rotary

TABLE 3

Comparison of Experimental and Computed Values of Stability Derivatives for Body without Fins

	Model 4164			Model 4166		
	Exper.	Laitone	Albring	Exper.	Laitone	Albring
Z_w	-.0101			-.0093		
M_w	.0132	.0118	.0133	.0178	.0175	.0198
Z_q	-.0011	-.0051	a) -.0025 b) 0	-.0014	-.0040	a) -.0029 b) 0
M_q	-.0008	-.0033	-.0007	-.0010	-.0025	-.0010

derivatives, the Laitone formulas give much poorer agreement. The experimental value of Z_q is almost exactly the mean of the two values computed by the Albring formulas given in References 3 and 4.

COMPARISON OF FORMULAS FOR BODY WITH FINS WITH EXPERIMENTAL VALUES

Four sets of formulas, Equations [7], [8], [9], and [11], have been given for the stability derivatives of a body with fins. The predictions from each of these will now be compared with the experimental results for Models 4164 and 4166.

In applying Equations [7] and [8] the experimental values given in Table 2 will be used for the bare-hull derivatives. The normal-force derivative Z_{wt} will be computed from the formula, derived from lifting-line theory with a small contribution of the drag of the fins neglected.

$$Z_{wt} = -\frac{2\pi a_t A_t}{a_t + 2} \qquad [43]$$

where a_t, A_t, and x_t are given in Table 1. The results obtained from Equations [7], and the results from Equations [8], with m taken from Table 1, are given in Table 4.

In applying Equations [9] the experimental values of Z_w and Z_{wo} for the body with and without fins, given in Table 2, will be used and L_w and L_{wo} computed from [6]. In using [6], the value of D need not be known with high accuracy since D is small in comparison with Z_w or Z_{wo}; D = 0.0010 may be assumed. The results from Equations [9] are also given in Table 4.

In Equations [11] the values of the bare-hull derivatives will again be taken from Table 2; A_t, m, a_t, and x_t from Table 1. On the basis of a rough analysis of Harrington's results,[13] a value $\gamma = 1.64$ will be assumed at a tail surface. Also taking $\lambda = 7.0$ gives $G = 1 - 25.6\ L_{wo}$ and $K = x_t - 2.56$ m. From Figure 1 and Appendix 3 we obtain ζ_1 and ζ_2; then F from [12]. ζ_1, ζ_2, F, G, and K are given in the following table:

	ζ_1	ζ_2	F	G	K
Model 4164	0.96	0.682	2.93	0.770	0.519
Model 4166	0.93	0.665	2.44	0.787	0.456

The results from Equations [11] are given in Table 4.

TABLE 4

Comparison of Experimental and Computed Values of Stability Derivatives for Body with Fins

	Experi-mental	Equations [7]	Simplest Formulas Equations [8]	Albring	New Formulas Equations [11]
			Model 4164		
Z_w	-.0200	-.0286	-.0185	-.0200 (assumed)	-.0194
M_w	.0079	.0028	.0084	.0106	.0079
Z_q	-.0106	-.0115	-.0092	a)-.0052 b)-.0027	-.0074
M_q	-.0053	-.0067	-.0046	-.0014	-.0043
			Model 4166		
Z_w	-.0210	-.0331	-.0238	-.0210 (assumed)	-.0208
M_w	.0117	.0056	.0105	.0132	.0119
Z_q	-.0098	-.0136	-.0119	a)-.0069 b)-.0040	-.0081
M_q	-.0053	-.0073	-.0060	-.0024	-.0044

It is seen from Table 4 that Albring's formulas give poor agreement with experiment for the rotary derivatives. Equations [7] are in poor agreement for the static derivatives. Both the "simplest formulas" [8] and the new formulas [11] give good agreement with the experimental values, and either set of formulas should serve as approximations to the stability derivatives.

It should be noted that the values from the new formulas are algebraically greater than (and, in one case, equal to) the experimental values for both models. This suggests that the agreement could have been improved by choosing a smaller value for the downwash parameter γ and larger values for the wake factor ζ_2. However, considering the possible errors in the experimental values themselves, no attempt will be made now to adjust the presently assumed values. Rather, it is suggested that such an empirical adjustment await the collection of additional reliable data.

SUMMARY

A review of existing theory indicates that a satisfactory theory for predicting the dynamic-stability derivatives of a body of revolution without fins has not yet been developed. Instead, the following empirical formulas appear to be best available:

$$Z_{wo} = -(0.23\, m^{0.79} + D) \tag{27}$$

$$M_{wo} = 0.87\,(k_2 - k_1)\, m \tag{29}$$

$$Z_{qo} = -(0.10 - k_1)\, m \tag{40}$$

$$M_{qo} = -0.045\, m \tag{41}$$

When the body is equipped with tail fins adequate to give it dynamic stability, the simplest set of prediction formulas [8], which are essentially those that have long been used by airship designers, appears to be in good agreement with experiment. These formulas are:

$$\left.\begin{aligned} &(a)\quad Z_w = Z_{wt} \\ &(b)\quad M_w = m + \tfrac{1}{2} Z_w \\ &(c)\quad Z_q = \tfrac{1}{2} Z_w \\ &(d)\quad M_q = \tfrac{1}{4} Z_w \end{aligned}\right\} \tag{8}$$

These formulas are convenient for many applications, such as are illustrated in Appendices 1 and 2. Because of the crudity of the assumptions upon which they are based, however, it is believed that the agreement of the predictions from Equations [8] with experiment is fortuitous. It is believed that for a model equipped with smaller fins, or with fins of lower aspect ratio than was considered here, so that interference effects between body and fins would become much more severe, the predictions would not be in good agreement with experiment.

The predictions from formulas [11] also appear to be in good agreement with experiment. In the derivation of these formulas it was attempted to take into account the interference between the hull and the tail. These formulas are:

$$
\begin{aligned}
&(a)\quad Z_w = Z_{wo} - FA_tG \\
&(b)\quad M_w = M_{wo} - x_tFA_tG \\
&(c)\quad Z_q = Z_{qo} - FA_tK \\
&(d)\quad M_q = M_{qo} - x_tFA_tK
\end{aligned}
\qquad [11]
$$

where

$$F = \frac{2\pi\zeta_1\zeta_2}{1 + 2/a_t} \qquad [12]$$

$$G = 1 - \frac{1}{\pi}\gamma\lambda^2 L_{wo} \qquad [13]$$

$$K = x_t - \frac{0.10}{\pi}\gamma\lambda^2 m \qquad [14]$$

and

$$Z_{wo} = -(L_{wo} + D) \qquad [6]$$

APPENDIX 1

ANALYSIS OF ERRORS IN ROOTS OF STABILITY EQUATION

In the present section the simplest approximation formulas [8] will be applied to obtain simple expressions and graphs for the errors in the roots of the stability equation, [1], due to errors in the values of the stability derivatives. Since, as was seen in Table 4, the formulas [8] give results in very good agreement with experiment for bodies of revolution with tail surfaces, it should be possible to determine at least the order of magnitude of these errors by the use of these formulas.

Inserting the values given in Equations [2], and the approximations $k' = k_2 = 1$, Equation [1] may be written

$$4m^2c^2\sigma^2 - 2m(c^2Z_w + M_q)\sigma + Z_wM_q - M_w(m + Z_q) = 0 \qquad [44]$$

where $c = \sqrt{I/m}$ is the radius of gyration about a transverse axis. The following expressions for error rates can now be obtained by differentiating [44]:

$$\begin{aligned} &(a) \quad \frac{\partial\sigma}{\partial Z_w} = (2mc^2\sigma - M_q)/\Delta_1 \\ &(b) \quad \frac{\partial\sigma}{\partial M_w} = (m + Z_q)/\Delta_1 \\ &(c) \quad \frac{\partial\sigma}{\partial Z_q} = M_w/\Delta_1 \\ &(d) \quad \frac{\partial\sigma}{\partial M_q} = (2m\sigma - Z_w)/\Delta_1 \end{aligned} \qquad [45]$$

where

$$\Delta_1 = 2m(4mc^2\sigma - c^2Z_w - M_q) \qquad [46]$$

If now the approximate relations [8] are assumed and we put $\mu = - Z_w/m$, we obtain

$$\begin{aligned} M_w &= m(-\mu + 2)/2 \\ Z_q &= - m\,\mu/2 \\ M_q &= - m\,\mu/4 \end{aligned} \qquad [47]$$

and also Equation [44] becomes

$$8c^2\sigma^2 + \mu(1 + 4c^2)\sigma + 2(\mu - 1) = 0$$

whence

$$-\frac{Z_w}{m} = \mu = \frac{2(1 - 4c^2\sigma^2)}{(1 + 4c^2)\sigma + 2} \quad [48]$$

Equations [47] and [48] enable the derivatives of σ, given by [45], to be written explicitly as functions of σ. We obtain

$$\begin{aligned}
&(a) \quad Z_w \frac{\partial\sigma}{\partial Z_w} = -(1 - 4c^2\sigma^2)(1 + 4c^2\sigma)^2/\Delta_2 \\
&(b) \quad M_w \frac{\partial\sigma}{\partial M_w} = [1 + (1 + 4c^2)\sigma + 4c^2\sigma^2]^2/\Delta_2 \\
&(c) \quad Z_q \frac{\partial\sigma}{\partial Z_q} = -(1 - 4c^2\sigma^2)[1 + (1 + 4c^2)\sigma + 4c^2\sigma^2]/\Delta_2 \\
&(d) \quad M_q \frac{\partial\sigma}{\partial M_q} = -(1 - 4c^2\sigma^2)(1 + \sigma)^2/\Delta_2
\end{aligned} \quad [49]$$

where

$$\Delta_2 = [2 + (1 + 4c^2)\sigma][1 + 4c^2 + 16c^2\sigma + 4c^2(1 + 4c^2)\sigma^2] \quad [50]$$

The error rates in Equations [49] are graphed against σ in Figure 5 for the case $c^2 = 0.05$, since this is approximately the value for most cases of practical interest. It is seen that the stability index σ is most sensitive to percentage changes in Z_w for a very stable body, in M_w for a very unstable body. For a body of neutral stability, ($\sigma = 0$), σ is equally sensitive to percentage changes in all the stability derivatives, i. e.,

$$-Z_w \frac{\partial\sigma}{\partial Z_w} = M_w \frac{\partial\sigma}{\partial M_w} = -Z_q \frac{\partial\sigma}{\partial Z_q} = -M_q \frac{\partial\sigma}{\partial M_q} = 0.417 \quad [51]$$

The latter value may also be assumed to give the order of magnitude of the error rates for nearly neutrally stable bodies. Thus, for such a body, a percentage error of 10 percent in one of the stability derivatives would result in an error $\Delta\sigma = 0.10 \times 0.417 = 0.042$.

For a body of neutral static stability, i.e., when $M_w = 0$, it is seen from Equation [45b] that $\partial\sigma/\partial M_w = 0$, so that σ is insensitive to small changes in M_w.

As an application of Figure 5, let us find the change in σ due to a change in the effective area of tail surface A_t, assuming that the effective aspect ratio a_t is held constant. We have

$$\frac{d\sigma}{dA_t} = \frac{\partial\sigma}{\partial Z_w}\frac{dZ_w}{dA_t} + \frac{\partial\sigma}{\partial M_w}\frac{dM_w}{dA_t} + \frac{\partial\sigma}{\partial Z_q}\frac{dZ_q}{dA_t} + \frac{\partial\sigma}{\partial M_q}\frac{dM_q}{dA_t} \quad [52]$$

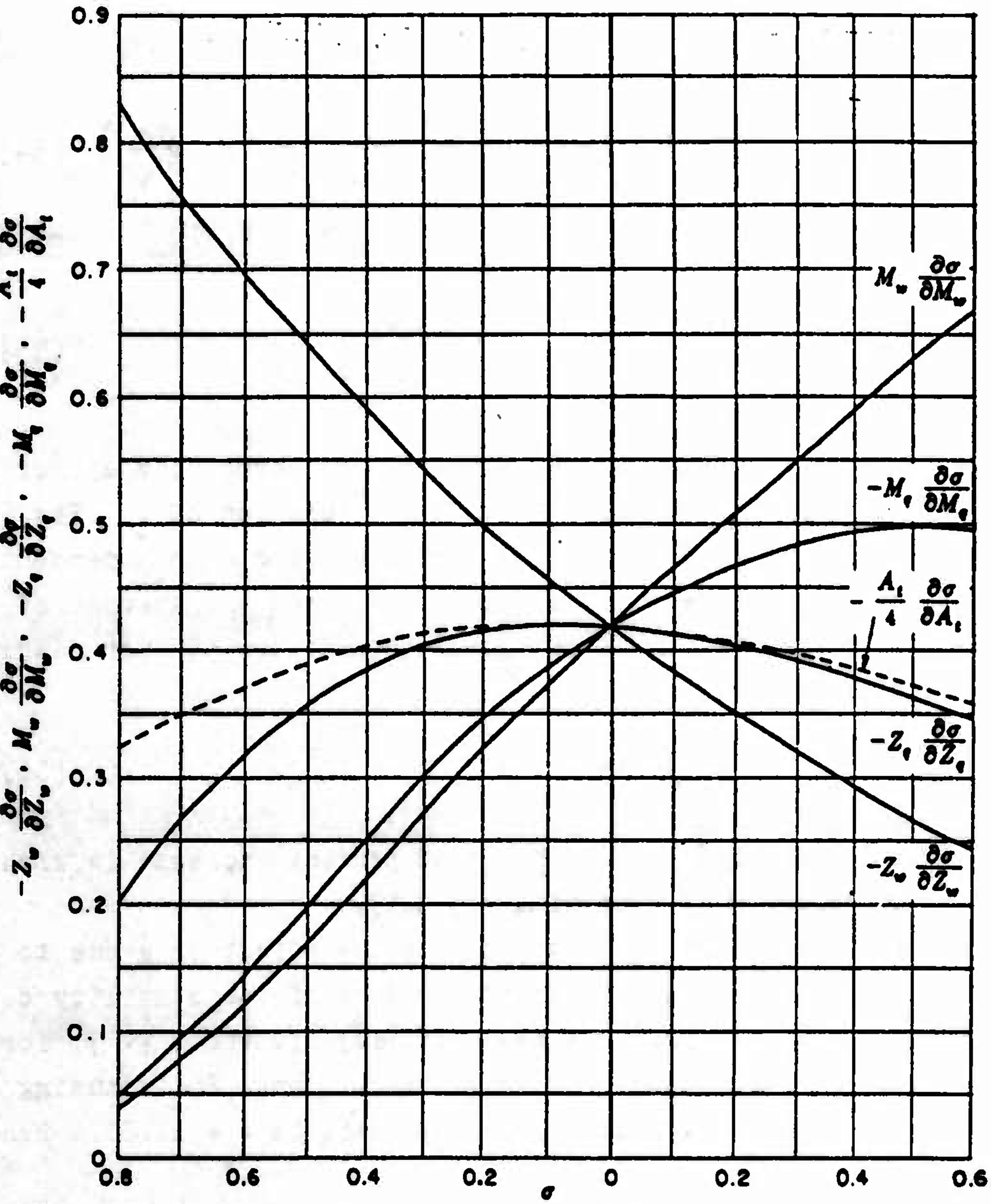

Figure 5 - Rates of Change in Stability Index Due to Percentage Changes in Stability Derivatives and Area

But from [8] and [43], in accordance with present approximations,

$$Z_w = Z_{wt} = -\frac{2\pi A_t}{1 + 2/a_t} \qquad [53]$$

and

$$\frac{1}{Z_w}\frac{dZ_w}{dA_t} = \frac{1}{Z_q}\frac{dZ_q}{dA_t} = \frac{1}{M_q}\frac{dM_q}{dA_t} = \frac{1}{A_t}$$

$$\frac{1}{M_w}\frac{dM_w}{dA_t} = \frac{1}{Z_w + 2m}\frac{dZ_w}{dA_t} = \frac{Z_w}{Z_w + 2m}\frac{1}{A_t}$$

Then [52] gives

$$A_t \frac{d\sigma}{dA_t} = \left(Z_w \frac{\partial\sigma}{\partial Z_w}\right) + \left(Z_q \frac{\partial\sigma}{\partial Z_q}\right) + \left(M_q \frac{\partial\sigma}{\partial M_q}\right) + \frac{Z_w}{Z_w + 2m}\left(M_w \frac{\partial\sigma}{\partial M_w}\right) \quad [54]$$

where, from [48]

$$\frac{Z_w}{Z_w + 2m} = \frac{\mu}{\mu - 2} = -\frac{1 - 4c^2\sigma^2}{1 + (1 + 4c^2)\sigma + 4c^2\sigma^2} \quad [55]$$

Since all the terms in parentheses in [54] are given in terms of σ either by Equations [49] or by Figure 5, then $A_t \frac{\partial\sigma}{\partial A_t}$ is also a function of σ. Its graph is included in Figure 5. The rate of variation of σ with a percentage change in tail-surface area is seen to be about four times the average of the rates of variation of the other quantities graphed in Figure 5. For nearly neutrally stable bodies we have

$$A_t \frac{\partial\sigma}{\partial A_t} = -1.67 \quad [56]$$

four times the value in Equation [51]. Thus a 10 percent increase in area of tail surface would reduce σ by approximately 0.17.

As another illustration, let us estimate the effect on σ due to the differences in Table 4 between the experimental values of the stability derivatives and the values computed from the new formulas, Equations [11], for Model 4166. The value of σ computed from Equations [1] and [2] (assuming $k' = k_2 = 1$) using the experimental data for Model 4166 is $\sigma = 0.122$. Hence, from Figure 5,

$$Z_w \frac{\partial\sigma}{\partial Z_w} = -0.375 \qquad M_w \frac{\partial\sigma}{\partial M_w} = 0.472$$

$$Z_q \frac{\partial\sigma}{\partial Z_q} = -0.412 \qquad M_q \frac{\partial\sigma}{\partial M_q} = -0.449$$

Also, from Table 4,

$$\frac{\Delta Z_w}{Z_w} = -0.010 \qquad \frac{\Delta M_w}{M_w} = 0.017$$

$$\frac{\Delta Z_q}{Z_q} = -0.173 \qquad \frac{\Delta M_q}{M_q} = -0.170$$

Hence,

$$\Delta\sigma = Z_w \frac{\partial\sigma}{\partial Z_w}\frac{\Delta Z_w}{Z_w} + M_w \frac{\partial\sigma}{\partial M_w}\frac{\Delta M_w}{M_w} + Z_q \frac{\partial\sigma}{\partial Z_q}\frac{\Delta Z_q}{Z_q} + M_q \frac{\partial\sigma}{\partial M_q}\frac{\Delta M_q}{M_q}$$

$$= .0038 + .0080 + .0712 + .0764 = 0.159$$

If we had used the less exact relations [51], assuming the body to be of approximately neutral stability, we would have obtained

$$\Delta\sigma = 0.417\ (0.010 + 0.017 + 0.173 + 0.170) = 0.154$$

The resulting estimated value of σ is then $\sigma = 0.122 + 0.159 = 0.281$

APPENDIX 2

SIZE OF STABILIZERS FOR A PRESCRIBED σ

As a second application the formulas for the stability derivatives will be applied to a practical design problem, to estimate the size of the stabilizing surfaces needed in order to attain a prescribed value of the stability index σ.

The tail surface area will first be computed on the basis of the simplest approximation formulas [8]. In keeping with the spirit of this approximation, the lift force on the tail will be assumed to be given by [19], uncorrected for the boundary-layer and downwash effects of the hull. Then, from [8] and [43]

$$Z_w = Z_{wt} = -\frac{2\pi A_t}{1 + 2/a_t} \qquad [57]$$

Hence, from [48],

$$\frac{2\pi A_t}{1 + 2/a_t} = \frac{2m(1 - 4c^2\sigma^2)}{(1 + 4c^2)\sigma + 2} \qquad [58]$$

When σ is prescribed, the right member of Equation [58] is given. Denoting it by C_1, putting

$$A_t = \frac{4b_t^2}{a_t} \qquad [59]$$

and substituting for a_t and b_t from [21] and [22] into [58], we obtain

$$8\pi(b^2 - R^2) = C_1(a + 2) \qquad [60]$$

This last equation can be used to determine the aspect ratio a when the semi-span b is given, or vice-versa.

We can also obtain an expression for A_t by substituting the new formulas [11] for the stability derivatives into the stability equation [44]. The result is

$$FA_t = -\frac{4m^2c^2\sigma^2 - 2m\sigma(c^2Z_{wo} + M_{qo}) + Z_{wo}M_{qo} - M_{wo}(m + Z_{qo})}{2m\sigma(c^2G + x_tK) - GM_{qo} - x_tKZ_{wo} + x_tG(m + Z_{qo}) + KM_{wo}} \qquad [61]$$

When the bare-hull stability derivatives and the desired σ are prescribed, the right member of Equation [61] is given. Denoting it by C_2, and substituting for F from [12], Equation [61] becomes

$$2\pi\zeta_1\zeta_2 A_t = C_2\left(1 + \frac{2}{a_t}\right) \qquad [62]$$

Hence, substituting for a_t, b_t, and A_t from [21], [22] and [59] into [61], we obtain

$$8\pi\zeta_2(b^2 - R^2) = C_2 \frac{a + 2}{\zeta_1} \qquad [63]$$

But ζ_1 is given as a function of a by Equation [20] or by Figure 1, and ζ_2 is given as a function of b in Appendix 3 (Equation [73]). Hence, Equation [63] can be used to determine a when b is given, or vice-versa.

As an example, consider Model 4164 with σ = -0.25 prescribed. This value of σ corresponds to the experimental values in Table 2, when it is computed from Equation [44] with c^2 = 0.05. In order to determine C_1 and C_2 we need the values of m, x_t, Z_{wo}, M_{wo}, Z_{qo}, M_{qo}, G, and K. These are given in Tables 1 and 2, and in the tabulation preceding Table 4. Substituting these values for Model 4164 in Equations [58] and [61], we obtain

$$C_1 = 0.0205, \qquad C_2 = 0.0148$$

Now suppose that the stabilizers are situated at the very end of the body (R = 0) and that an aspect ratio a_t = 4.93 is prescribed. We then obtain from [60]

$$b = \sqrt{\frac{C_1(a + 2)}{8\pi}} = 0.0752$$

To apply [63], we first read ζ_1 = 0.96 from Figure 1. Also, from [73], we have

$$\zeta_2 = 0.90\left(1 - \frac{0.01726}{b}\right)$$

Hence, [63] becomes

$$b^2 - 0.01726\, b - 0.00473 = 0$$

whose solution is b = 0.0779. The corresponding tail areas, computed from [59], are as follows:

Actual tail area	0.00414
Computed from "simple" formulas	0.00459
Computed from new formulas	0.00492

As another example, consider Model 4166 with σ = 0.122, corresponding to the experimental values in Table 2. Substituting the values from Tables 1 and 2, and G and K from the tabulation preceding Table 4 into Equations [58] and [61], we obtain

$$C_1 = 0.0208, \qquad C_2 = 0.0168$$

Now suppose, in contrast to the previous example, that b = 1/14 is prescribed, and that R = 0, as before. We then obtain, from [60],

$$a = \frac{8\pi b^2}{C_1} - 2 = 4.17$$

To apply [63] we first obtain $\zeta_2 = 0.665$ from [73]. Equation [63] may now be solved by successive approximations. Assume $\zeta_1 = 0.95$; then, from Equation [63] we obtain, as a first approximation to a

$$a = \frac{8\pi\zeta_1\zeta_2 b^2}{C_2} - 2 = 2.82$$

The corrected value for ζ_1 corresponding to this value of a is $\zeta_1 = 0.91$. The second approximation to a is then a = 2.62, to which corresponds $\zeta_1 = 0.90$. We obtain, finally, a = 2.57. The corresponding tail areas, computed from [59], are as follows:

Actual tail area	0.00602
Computed from "simple" formulas	0.00490
Computed from new formulas	0.00794

Because of the possibility of errors in the experimental values, an absolute evaluation of the validity of these area formulas cannot be based on the foregoing comparisons.

APPENDIX 3

DETERMINATION OF WAKE FACTOR ζ_2

It will be convenient to employ dimensional nomenclature in the present section. Let U be the free-stream velocity, and at first assume that this is also the velocity just outside the boundary layer in the neighborhood of the tail surface. Let u be the longitudinal component of the velocity at a distance y from the body. Then, assuming rectangular loading on the tail fin, the wake correction factor ζ_2 is

$$\zeta_2 = \frac{1}{b_o}\int_0^{b_o} \left(\frac{u}{U}\right)^2 dy \qquad [64]$$

where b_o is the distance of the tip of the tail fin from the body.

It will be supposed that b_o is greater than the thickness of the boundary layer at the tail, so that we may write

$$\delta^* = \int_0^{b_o} \left(1 - \frac{u}{U}\right) dy \qquad [65]$$

and

$$\theta = \int_0^{b_o} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \qquad [66]$$

where δ^* is the displacement thickness and θ the momentum thickness of the boundary layer at the tail. Then

$$\delta^* + \theta = b_o - \int_0^{b_o} \left(\frac{u}{U}\right)^2 dy$$

whence, from [54],

$$\zeta_2 = 1 - \frac{\delta^* + \theta}{b_o} \qquad [67]$$

In terms of the boundary layer shape parameter $H = \delta^*/\theta$, Equation [67] becomes

$$\zeta_2 = 1 - \frac{\theta}{b_o}(H + 1) \qquad [68]$$

According to Granville and Lyon,[28,29] an approximate value for θ/b_o at the tail is given by

$$\left(\frac{\theta}{b_o}\right)^2 = \frac{(H - 1)(H + 3)}{2.8H^2(H + 1)} \lambda \left(\frac{d}{b_o}\right)^2 C_s C_t \qquad [69]$$

where d is the diameter of the body. Hence

$$\zeta_2 = 1 - \frac{0.6}{H}\frac{d}{b_o}\sqrt{(H^2 - 1)(H + 3)\lambda C_s C_t} \qquad [70]$$

Equation [70] has been derived on the assumption that the velocity just outside the boundary layer is equal to the free-stream velocity. Pressure distribution data on bodies of revolution, however, indicate that the velocity just outside the boundary layer at the tail is about 0.9U. When the fin extends well beyond the boundary layer, the mean potential-flow velocity over the fin will be somewhat larger than 0.9U. An average value of 0.95U will be assumed here, so that the wake factor ζ_2 becomes

$$\zeta_2 = 0.90 \left(1 - \frac{0.6}{H} \frac{d}{b_o} \sqrt{(H^2 - 1)(H + 3)\lambda C_s C_t}\right) \qquad [71]$$

If also the average value H = 1.6 is assumed, [71] becomes

$$\zeta_2 = 0.90 \left(1 - \frac{d}{b_o} \sqrt{\lambda C_s C_t}\right) \qquad [72]$$

This can also be expressed in our customary dimensionless notation, as

$$\zeta_2 = 0.90\left(1 - \frac{1}{b - H} \sqrt{\frac{C_s C_t}{\lambda}}\right) \qquad [73]$$

For Models 4164 and 4166, $\lambda = 7$, $d/b_o = 2$, and the values of C_s are given in Table 1. Assuming $C_t = 0.003$, we obtain the following values of ζ_2:

Model 4164, $\zeta_2 = 0.682$

Model 4166, $\zeta_2 = 0.665$

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